Missing data

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Missing data mechanisms

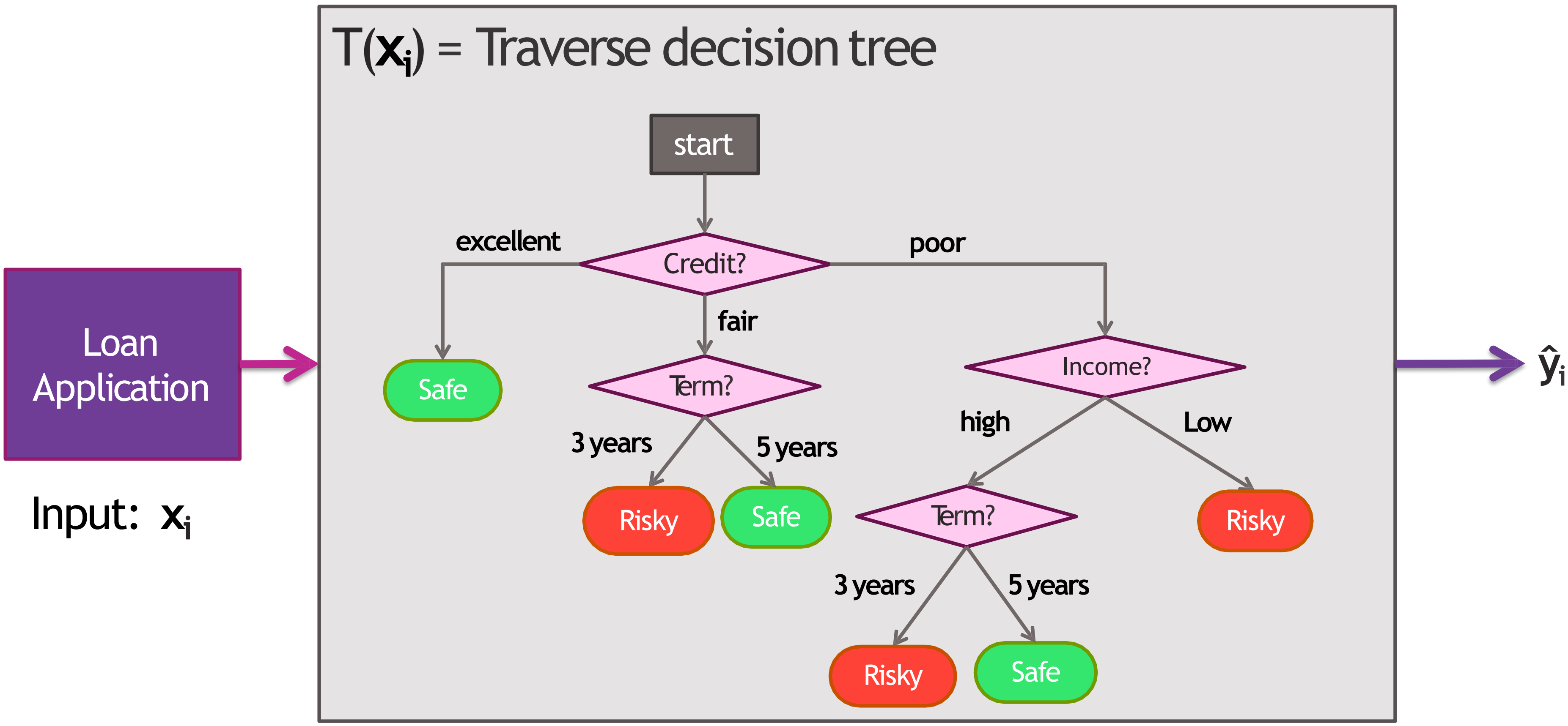
Unrealistic

- Missingness completely at random
  - A variable is missing completely at random if the probability of missingness is the same for all units, for example, if each survey respondent decides whether to answer the “earnings” question by rolling a die and refusing to answer if a “6” shows up. If data are missing completely at random, then throwing out cases with missing data does not bias your inferences.
- Missingness at random (ignorability assumption)
  - The probability a variable is missing depends only on available information. Thus, if sex, race, education, and age are recorded for all the people in the survey, then “earnings” is missing at random if the probability of nonresponse to this question depends only on these other, fully recorded variables

More realistic

- Missingness that depends on the unobserved covariates
  - Missingness is no longer “at random” if it depends on information that has not been recorded and this information also predicts the missing values. A familiar example from medical studies is that if a particular treatment causes discomfort, a patient is more likely to drop out of the study. If missingness is not at random, it must be explicitly modeled, or else you must accept some bias in your inferences
- Missingness that depends on the missing value itself (censoring)
  - The probability of missingness depends on the (potentially missing) variable itself. For example, suppose that people with higher earnings are less likely to reveal them.

Decision tree review



So far: data always completely observed

Credit	Term	Income	y
excellent	3 yrs	high	safe
fair	5 yrs	low	risky
fair	3 yrs	high	safe
poor	5 yrs	high	risky
excellent	3 yrs	low	risky
fair	5 yrs	low	safe
poor	3 yrs	high	risky
poor	5 yrs	low	safe
fair	3 yrs	high	safe

Known x and y values for all data points

Missing data

Credit	Term	Income	y
excellent	3 yrs	high	safe
fair	?	low	risky
fair	3 yrs	high	safe
poor	5 yrs	high	risky
excellent	3 yrs	low	risky
fair	5 yrs	high	safe
poor	?	high	risky
poor	5 yrs	low	safe
fair	?	high	safe

Loan application
may be
3 or 5 years

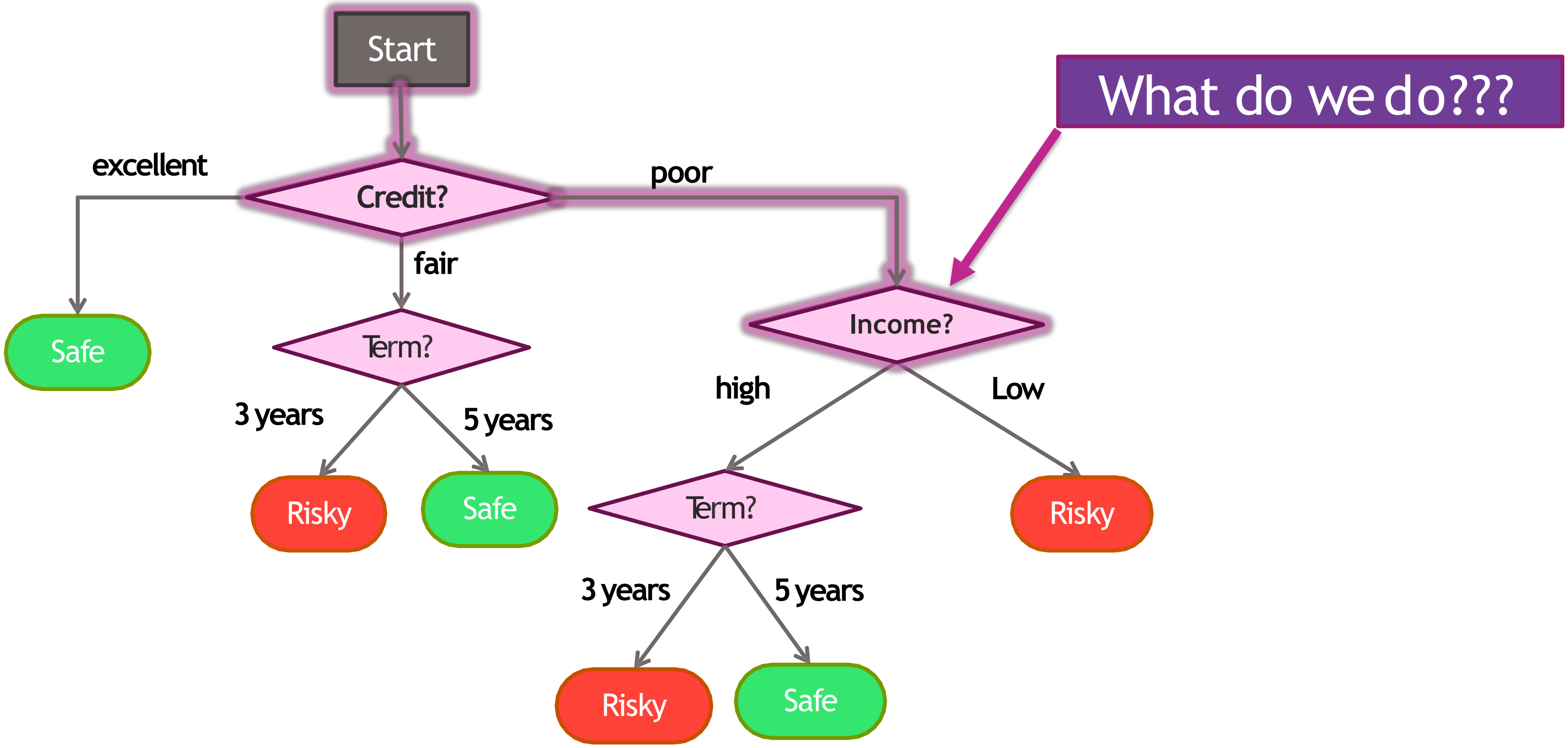
Missing values impact training and predictions

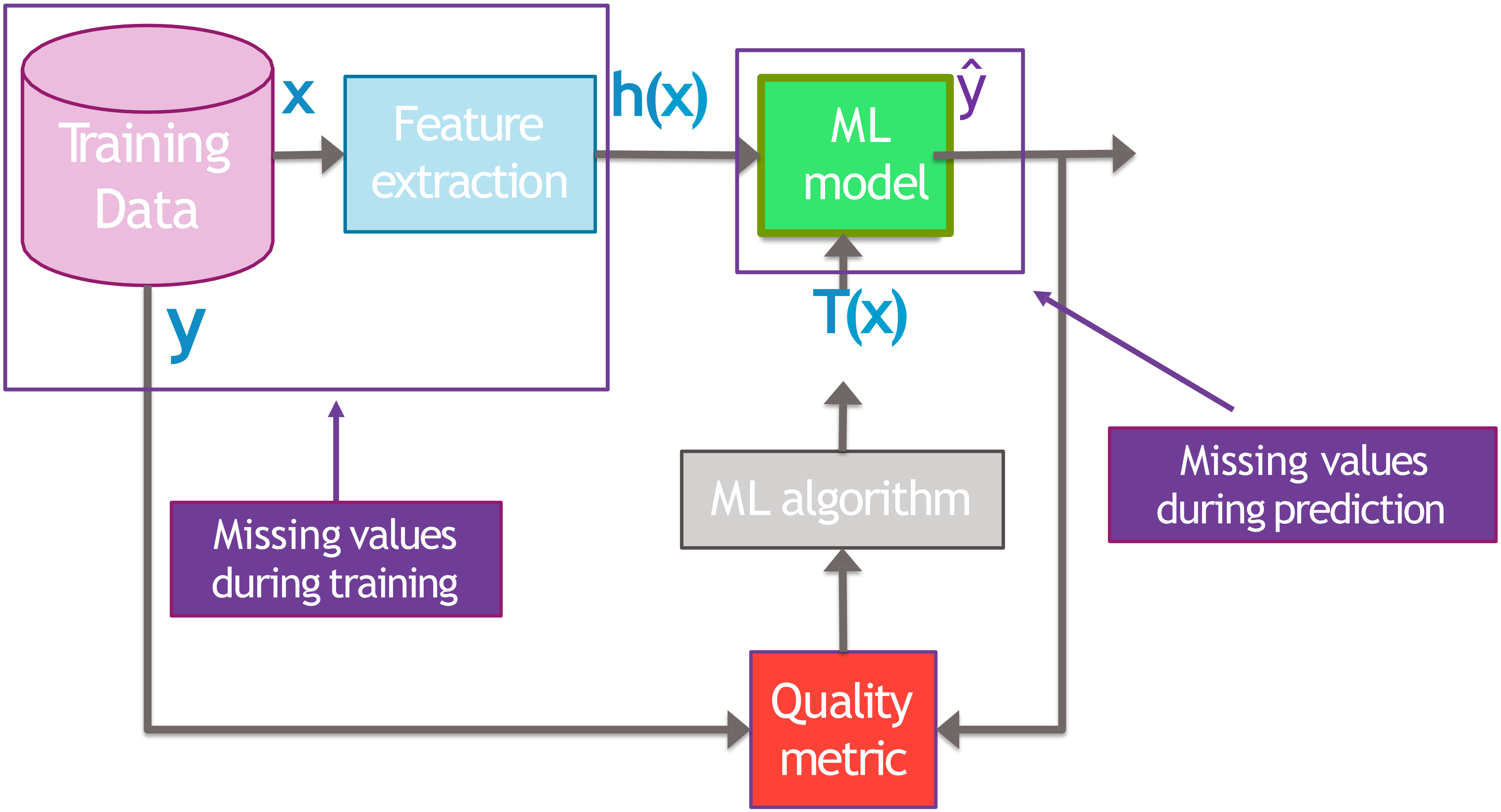
1. Training data: Contains “unknown” values

2. Predictions: Input at prediction time contains “unknown” values

Missing values during prediction

x_i = (Credit = poor, Income = ?, Term = 5 years)

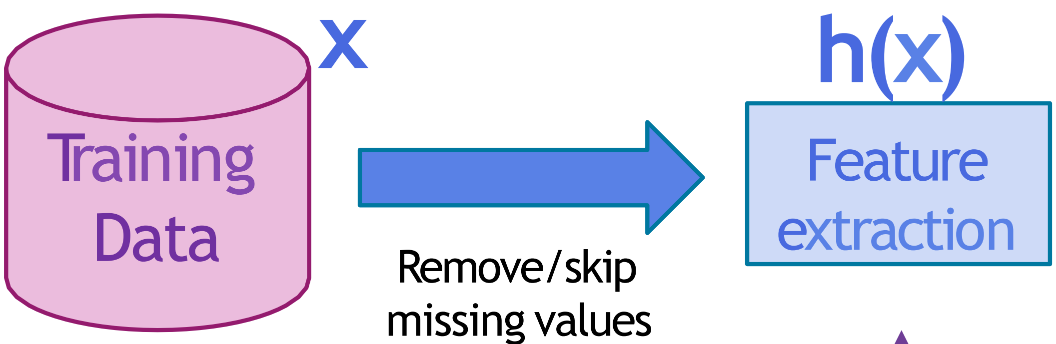



Training
Data
x
Feature
extraction
h(x)
ML
model
ŷ
y
T(x)
ML algorithm
Missing values
during training
Missing values
during prediction
Quality
metric

Handling missing data

Strategy 1: Purification by skipping

Idea 1: Purification by skipping/removing



May contain datapoints where one (or more) features are missing

h(x) is a subset of the data **without any missing** values

Idea 1: Skip data points with missing values

x

N = 9, 3 features

fewer

Credit	Term	Income	y
excellent	3 yrs	high	safe
fair	?	low	risky
fair	3 yrs	high	safe
poor	5 yrs	high	risky
excellent	3 yrs	low	risky
fair	5 yrs	high	safe
poor	3 yrs	low	risky
poor	3 yrs	?	safe
fair	?	high	safe

Skip data points with missing values

h(x)

N = 6, 3 features

Credit	Term	Income	y
excellent	3 yrs	high	safe
fair	3 yrs	high	safe
poor	5 yrs	high	risky
excellent	3 yrs	low	risky
fair	5 yrs	high	safe
poor	3 yrs	low	risky

The challenge with Idea 1

x

N = 9, 3 features

Credit	Term	Income	y
excellent	3 yrs	high	safe
fair	?	low	risky
fair	3 yrs	high	safe
poor	?	high	risky
excellent	?	low	risky
fair	?	high	safe
poor	3 yrs	low	risky
poor	?	low	safe
fair	?	high	safe

Warning: More than 50% of the loan terms are unknown!

much much smaller

Skip data points with missing values

h(x)

N = 3, 3 features

Credit	Term	Income	y
excellent	3 yrs	high	safe
fair	3 yrs	high	safe
poor	3 yrs	low	risky

Idea 2: Skip features with missing values

x

N = 9, 3 features

Credit	Term	Income	y
excellent	3 yrs	high	safe
fair	?	low	risky
fair	3 yrs	high	safe
poor	?	high	risky
excellent	?	low	risky
fair	5 yrs	high	safe
poor	?	high	risky
poor	?	low	safe
fair	?	high	safe

fewer features

Skip features with many missing values

h(x)

N = 9, 2 features

Credit	Income	y
excellent	high	safe
fair	low	risky
fair	high	safe
poor	high	risky
excellent	low	risky
fair	high	safe
poor	high	risky
poor	low	safe
fair	high	safe

Missing value skipping: Ideas 1 & 2

Idea 1: Skip data points where any feature contains a missing value

- Make sure only a few data points are skipped

Idea 2: Skip an entire feature if it's missing for many data points

- Make sure only a few features are skipped

Missing value skipping: Pros and Cons

Pros

- Easy to understand and implement
- Can be applied to any model (decision trees, logistic regression, linear regression,…)

Cons

- Removing data points and features may remove important information from data
- Unclear when it's better to remove data points versus features
- Doesn't help if data is missing at prediction time

Handling missing data

Strategy 2: Prification by imputing

Main drawback of skipping strategy

x

N = 9, 3 features

Data is precious, don't throw it away!

Credit	Term	Income	y
excellent	3 yrs	high	safe
fair	?	low	risky
fair	3 yrs	high	safe
poor	5 yrs	high	risky
excellent	3 yrs	low	risky
fair	5 yrs	high	safe
poor	3 yrs	low	risky
poor	?	low	safe
fair	?	high	safe

Skip data points with missing values

h(x)

N = 6, 3 features

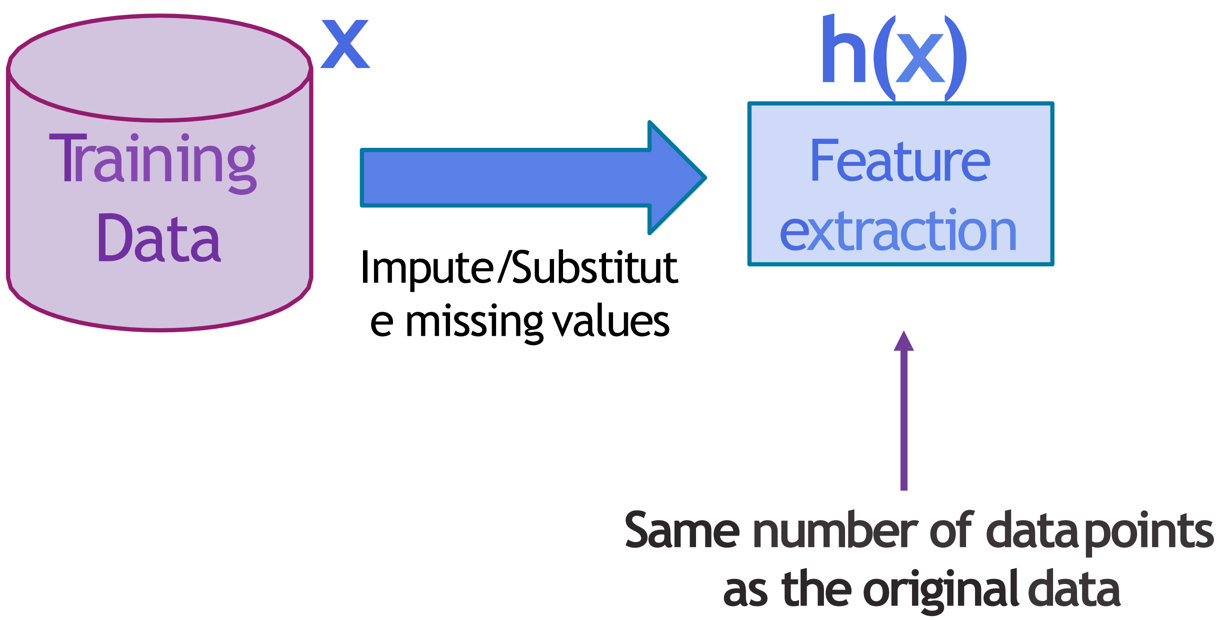
Credit	Term	Income	y
excellent	3 yrs	high	safe
fair	3 yrs	high	safe
poor	5 yrs	high	risky
excellent	3 yrs	low	risky
fair	5 yrs	high	safe
poor	3 yrs	low	risky

Can we keep all the data?

credit	term	income	y
excellent	3 yrs	high	safe
fair	?	low	risky
fair	3 yrs	high	safe
poor	5 yrs	high	risky
excellent	3 yrs	low	risky
fair	5 yrs	high	safe
poor	3 yrs	high	risky
poor	?	low	safe
fair	?	high	safe

Use other data points in **x** to "**guess**" the "?"

Idea 2: Purification by imputing



Idea 2: Imputation/Substitution

N = 9, 3 features

Credit	Term	Income	y
excellent	3 yrs	high	safe
fair	?	low	risky
fair	3 yrs	high	safe
poor	5 yrs	high	risky
excellent	3 yrs	low	risky
fair	5 yrs	high	safe
poor	3 yrs	high	risky
poor	?	low	safe
fair	?	high	safe

Fill in each missing value with a calculated guess

N = 9, 3 features

Credit	Term	Income	y
excellent	3 yrs	high	safe
fair	3 yrs	low	risky
fair	3 yrs	high	safe
poor	5 yrs	high	risky
excellent	3 yrs	low	risky
fair	5 yrs	high	safe
poor	3 yrs	high	risky
poor	3 yrs	low	safe
fair	3 yrs	high	safe

Example: Replace ? with most common value

\# 3year loans: 4 ← Best simple guess
\# 5year loans: 2

Credit	Term	Income	y
excellent	3 yrs	high	safe
fair	?	low	risky
fair	3 yrs	high	safe
poor	5 yrs	high	risky
excellent	3 yrs	low	risky
fair	5 yrs	high	safe
poor	3 yrs	high	risky
poor	?	low	safe
fair	?	high	safe

Purification by imputing

Credit	Term	Income	y
excellent	3 yrs	high	safe
fair	3 yrs	low	risky
fair	3 yrs	high	safe
poor	5 yrs	high	risky
excellent	3 yrs	low	risky
fair	5 yrs	high	safe
poor	3 yrs	high	risky
poor	3 yrs	low	safe
fair	3 yrs	high	safe

Common (simple) rules for purification by imputation

Credit	Term	Income	y
excellent	3 yrs	high	safe
fair	?	low	risky
fair	3 yrs	high	safe
poor	5 yrs	high	risky
excellent	3 yrs	low	risky
fair	5 yrs	high	safe
poor	3 yrs	high	risky
poor	?	low	safe
fair	?	high	safe

Impute each feature with missing values:

1. Categorical features use mode: Most popular value (mode) of non-missing x_i
2. Numerical features use average or median: Average or median value of non-missing x_i

Many advanced methods exist,
e.g., expectation-maximization (EM) algorithm

Missing value imputation: Pros and Cons

Pros

- Easy to understand and implement
- Can be applied to any model
  (decision trees, logistic regression, linear regression,…)
- Can be used at prediction time: use same imputation rules

Cons

- May result in systematic errors

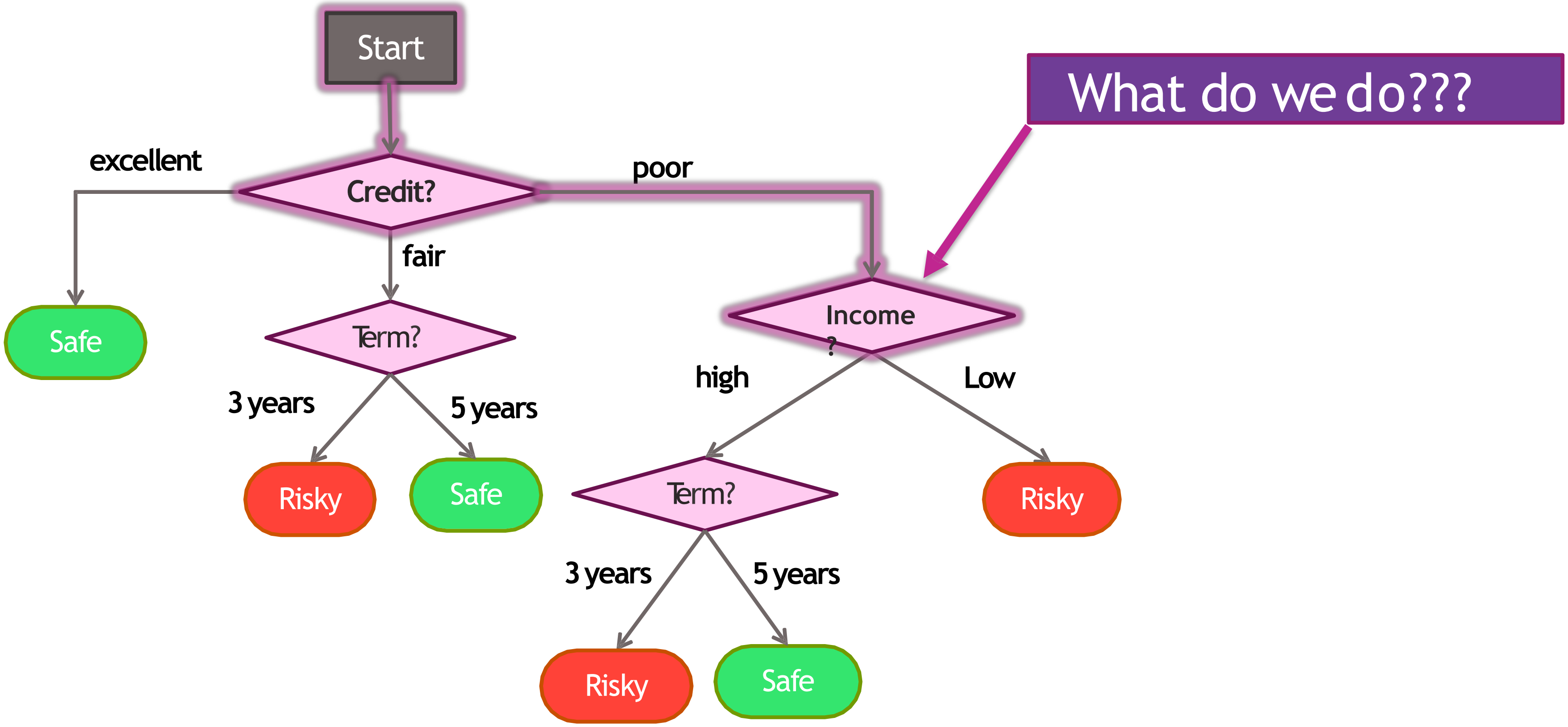
Example: Feature "age" missing in all banks in Washington by state law

Handling missing data

Strategy 3: Adapt learning algorithm to be robust to missing values

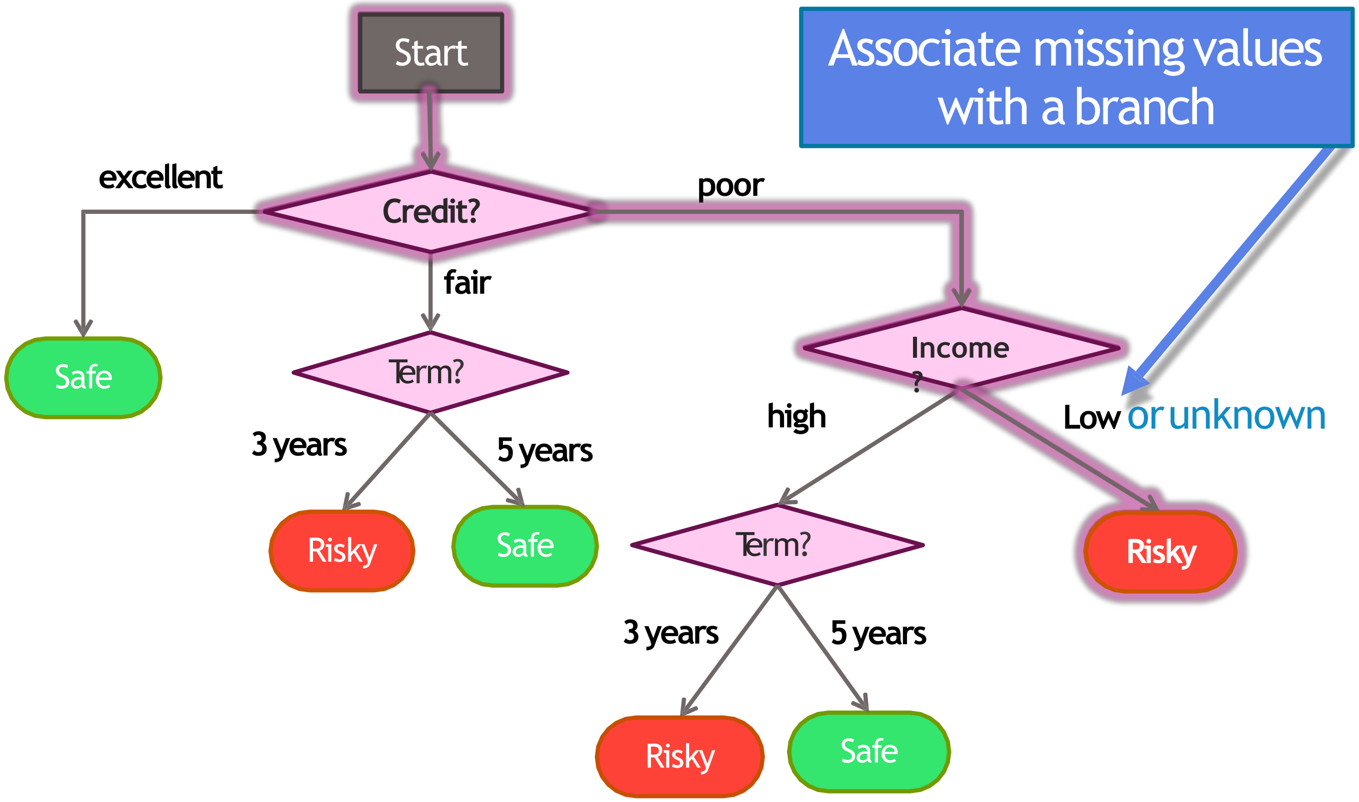
Missing values during prediction: *revisited*

x_i = (Credit = poor, Income = ?, Term = 5 years)

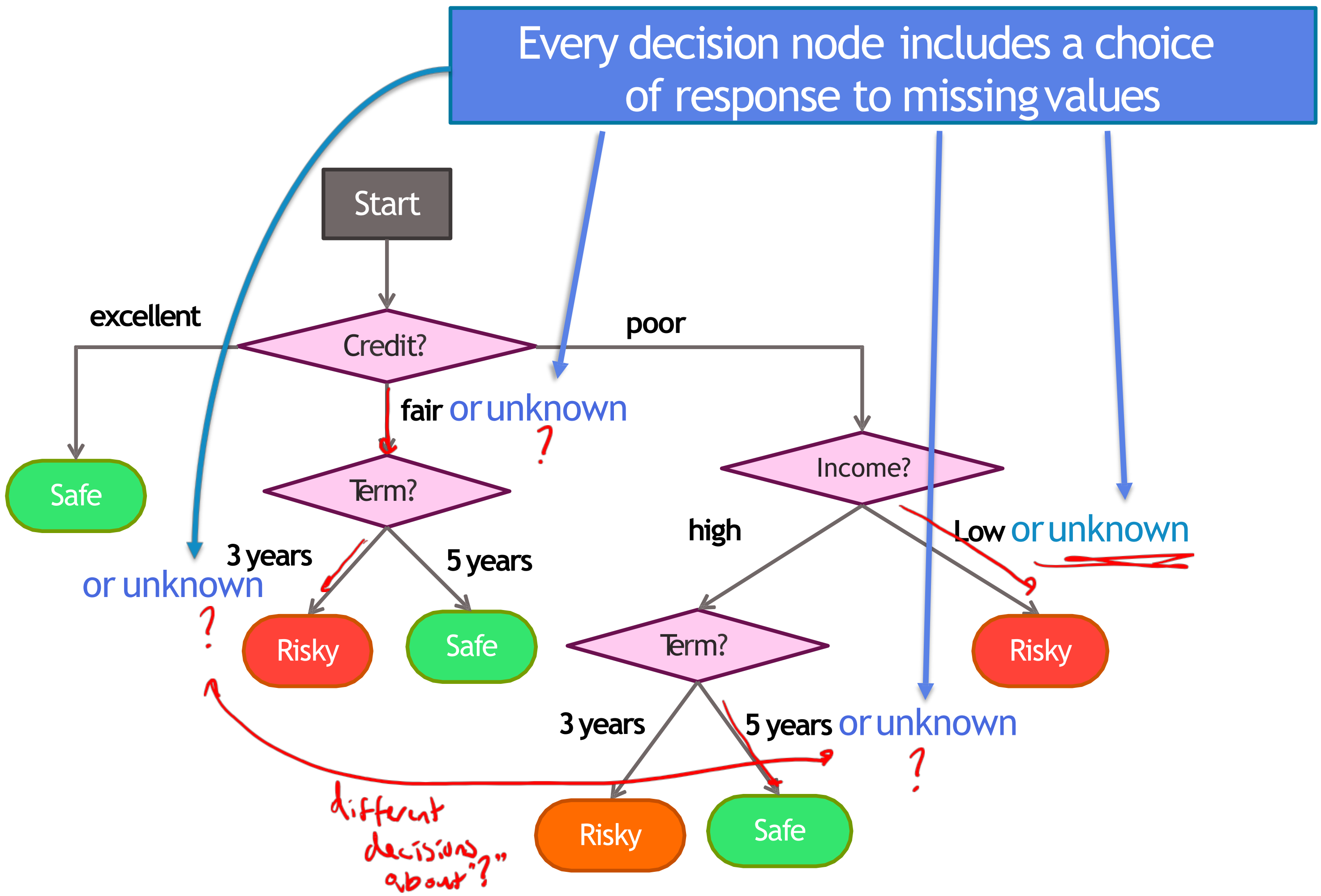


Add missing values to the tree definition

x_i = (Credit = poor, Income = ?, Term = 5 years)

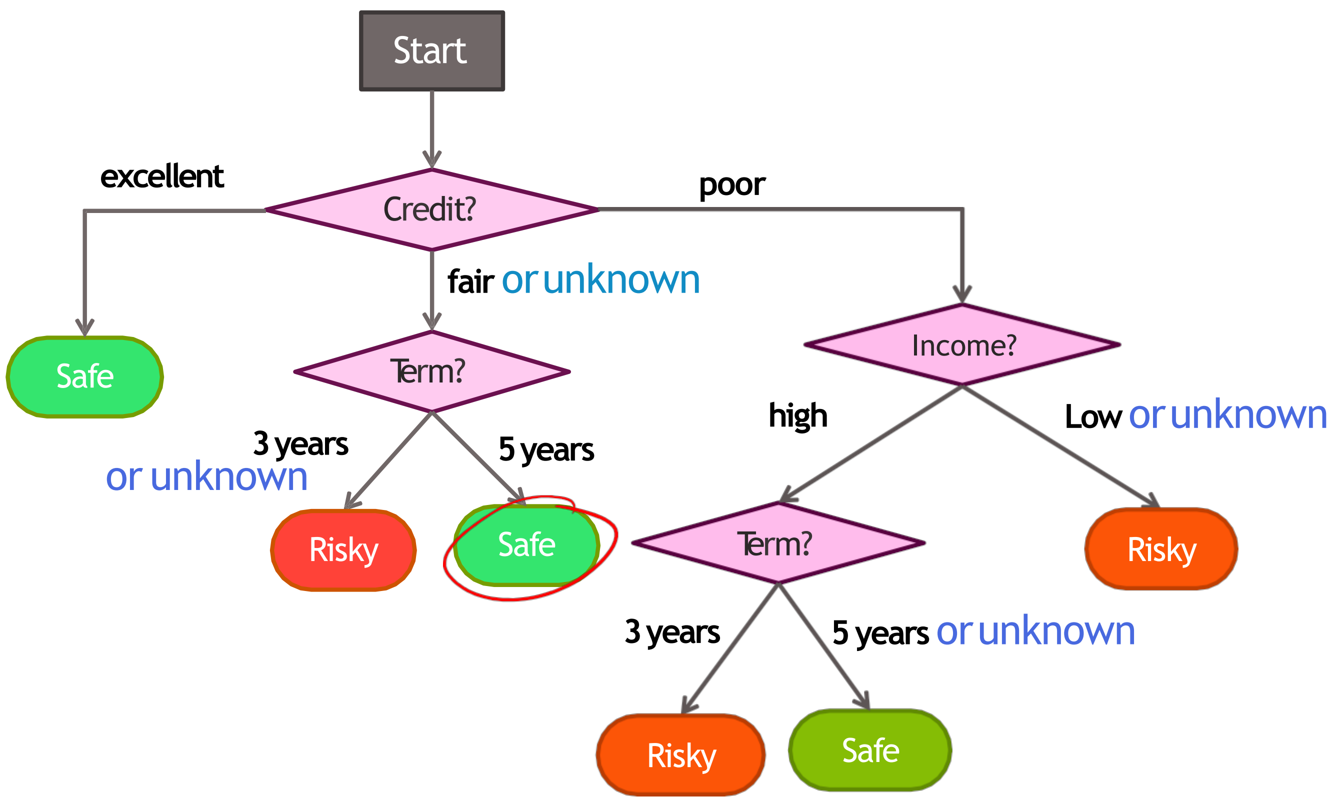


Add missing value choice to every decision node



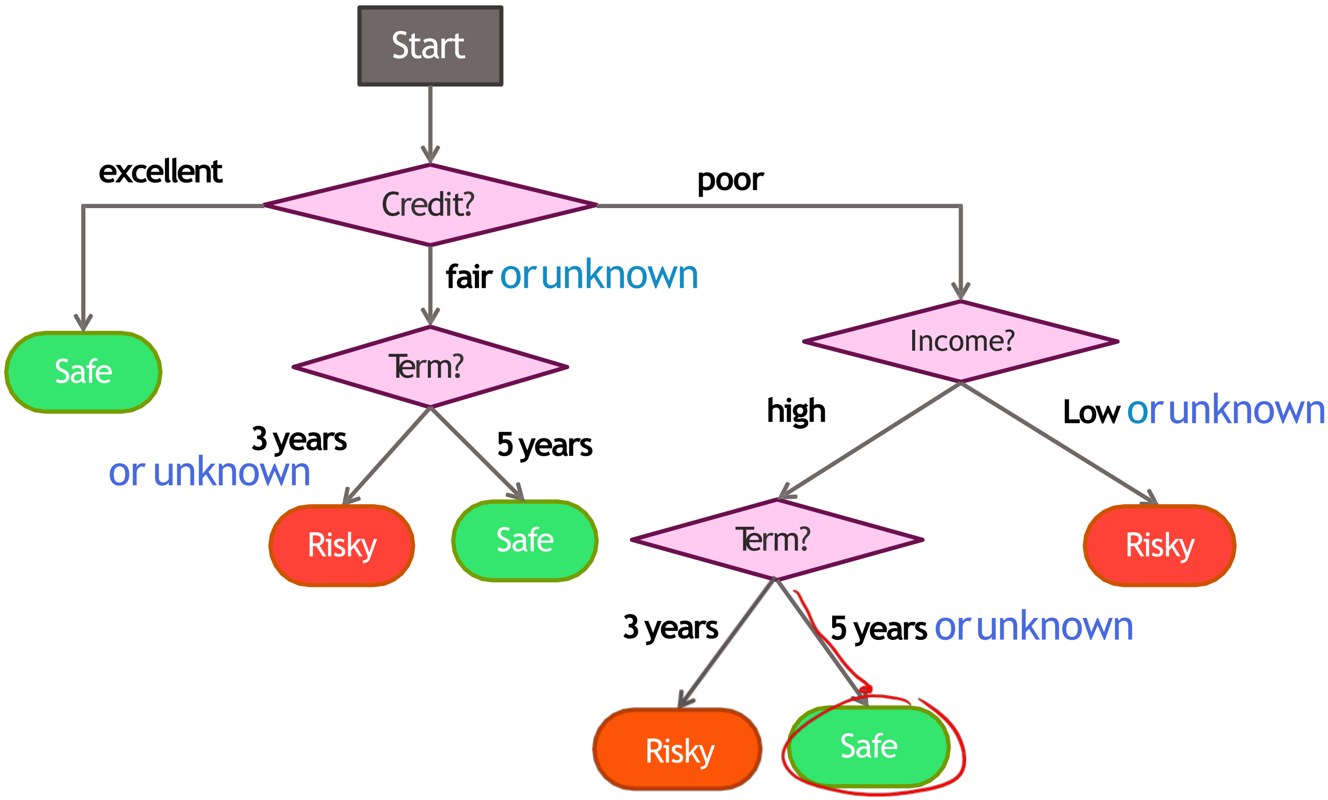
Prediction with missing values becomes simple

x_i = (Credit = ?, Income = high, Term = 5 years)



Prediction with missing values becomes simple

x_i = (Credit = poor, Income = high, Term = ?)



Explicitly handling missing data by learning algorithm: Pros and Cons

Pros

- Addresses training and prediction time
- More accurate predictions

Cons

- Requires modification of learning algorithm
  - Very simple for decision trees

Feature split selection with missing data

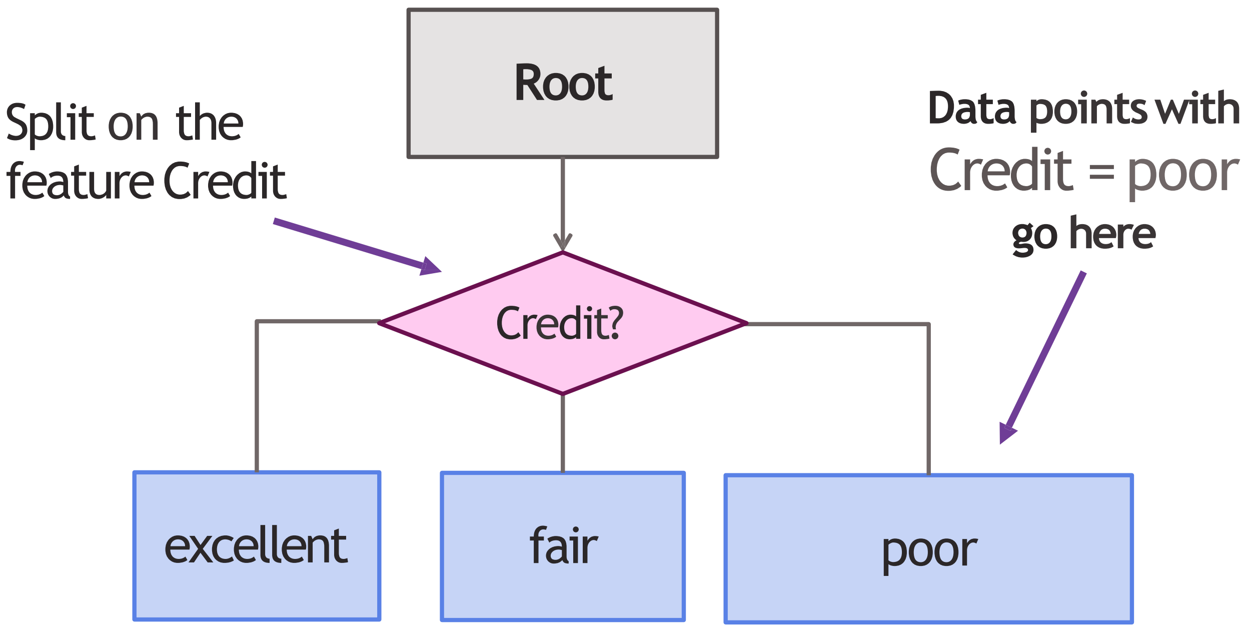
Greedy decision tree learning

- Step 1: Start with an empty tree
- Step 2: Select a feature to split data
- For each split of the tree:
  - Step 3: If nothing more to, make predictions
  - Step 4: Otherwise, go to Step 2 & continue (recurse) on this split

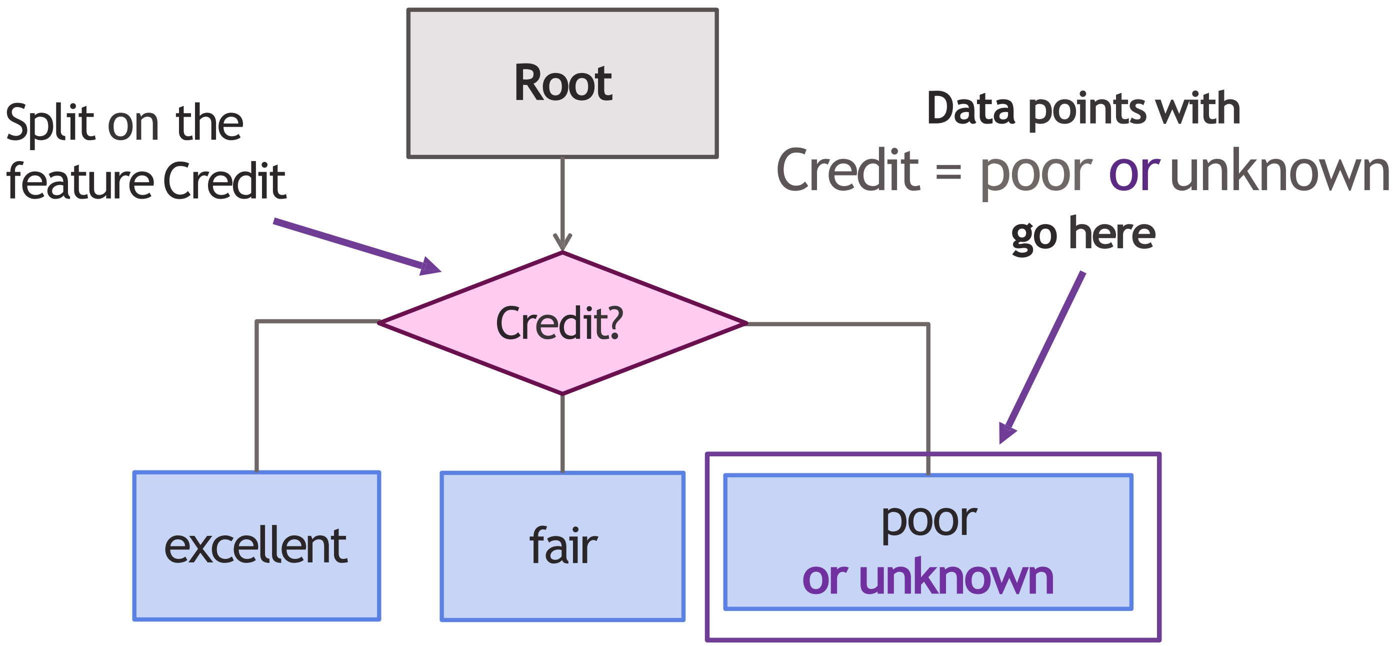
Pick feature split leading to lowest classification error

Must select feature & branch for missing values!

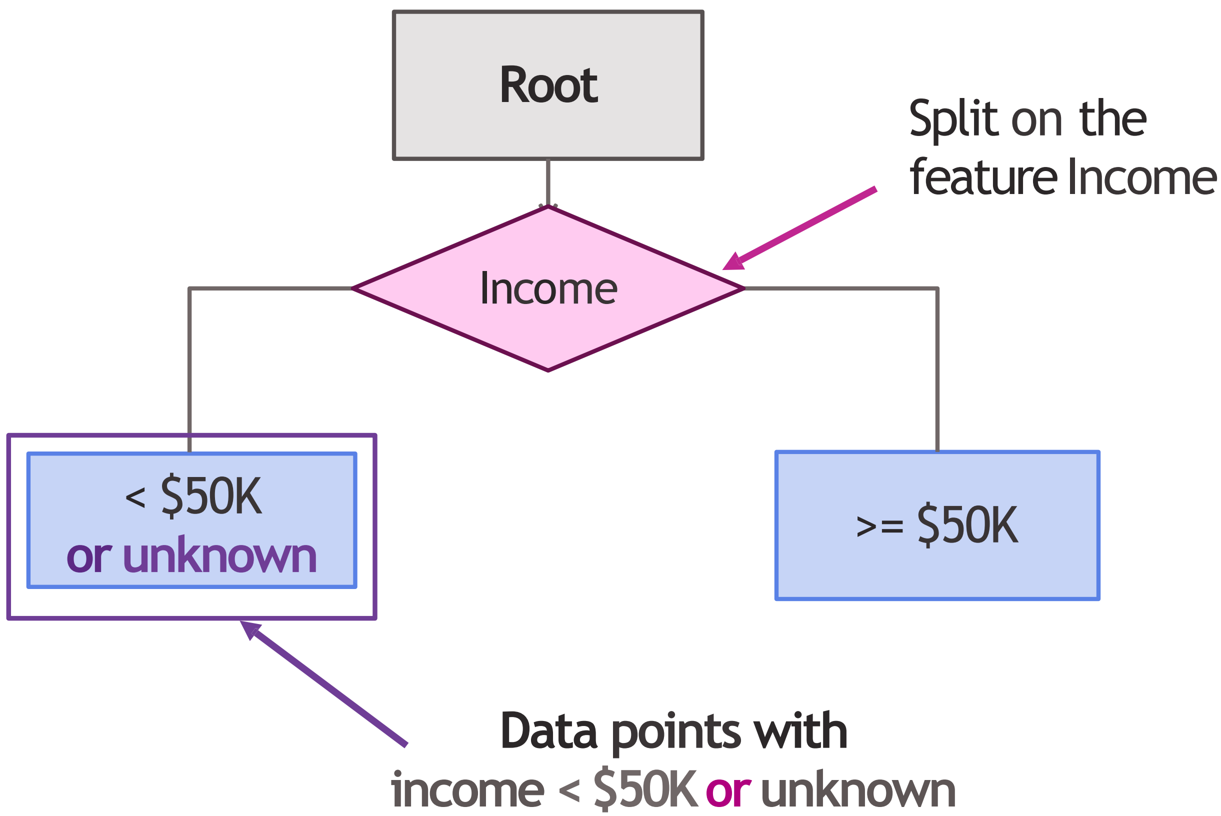
Feature split (without missing values)



Feature split (with missing values)



Missing value handling in threshold splits



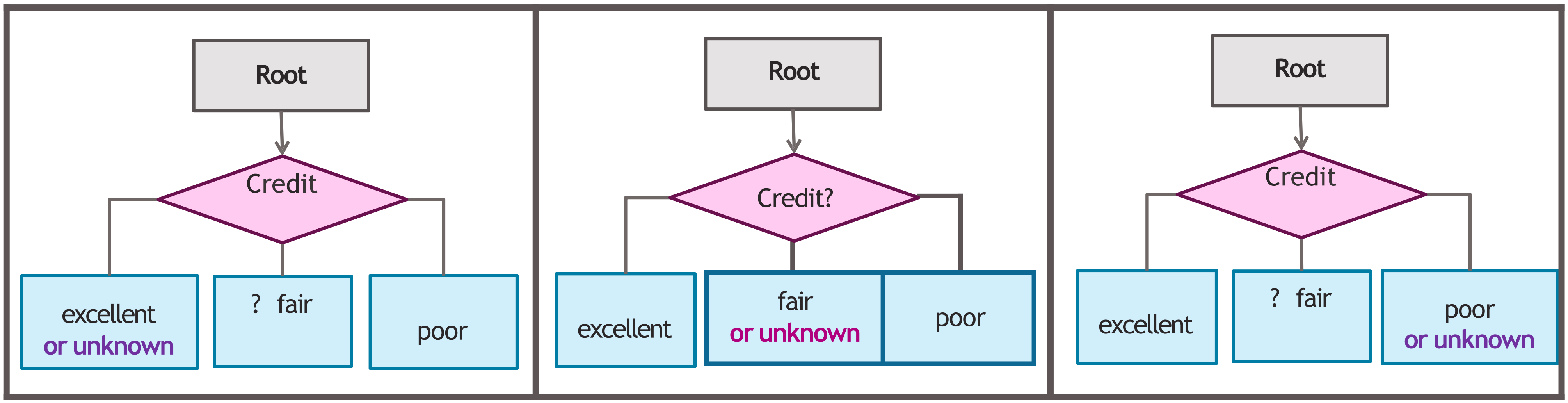
Should missing go left, right, or middle?

Choose branch that leads to lowest classification error!

Choice 1: Missing values go with Credit=excellent

Choice 2: Missing values go with Credit=fair

Choice 3: Missing values go with Credit=poor



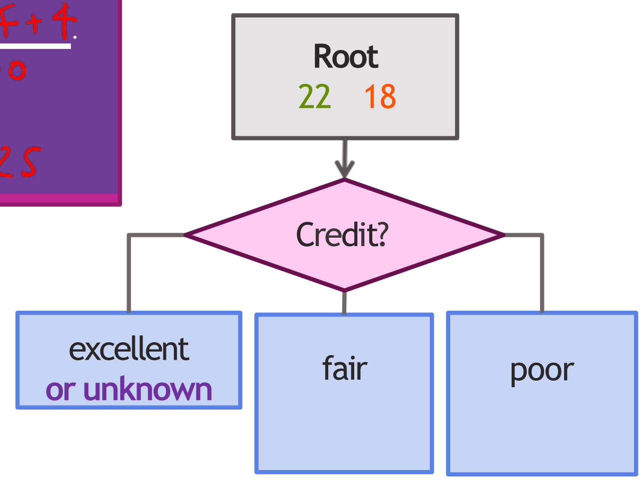
Computing classification error of decision stump with missing data

N = 40, 3 features

Credit	Term	Income	y
excellent	3 yrs	high	safe
?	5 yrs	low	risky
fair	3 yrs	high	safe
poor	5 yrs	high	risky
?	3 yrs	low	risky
?	5 yrs	low	safe
poor	3 yrs	high	risky
poor	5 yrs	low	safe
fair	3 yrs	high	safe
…	…	…	…

$$\text{Error} = \frac{2+4+4}{40}$$

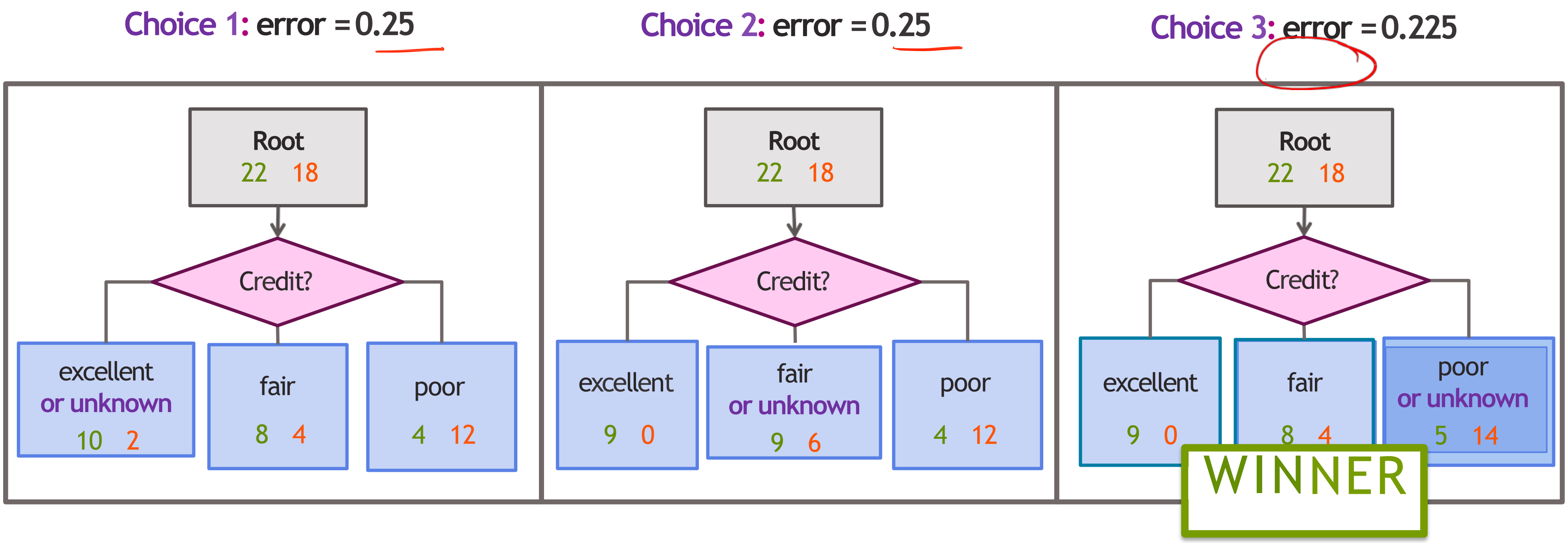
$$= 0.25$$



Observed values	9 0	8 4	4 12

Use classification error to decide

Best choice → assign "unknown" to Credit = poor



Feature split selection algorithm with missing value handling

- Given a subset of data M (a node in a tree)
- For each feature $h_i(x)$:
  1. Split data points of M where $h_i(x)$ is *not* "unknown" according to feature $h_i(x)$
  2. Consider assigning data points with "unknown" value for $h_i(x)$ to each branch
     A. Compute classification error split & branch assignment of "unknown" values
- Chose feature $h^*(x)$ & branch assignment of "unknown" with lowest classification error

Summary of handling missing data

What you can do now…

Describe common ways to handling missing data:

1. Skip all rows with any missing values
2. Skip features with many missing values
3. Impute missing values using other data points

Modify learning algorithm (**decision trees**) to handle missing data:

1. Missing values get added to one branch of split
2. Use classification error to determine where missing values go